A Monte Carlo Study of the Strategies for 401(k) Plans: Dollar-Cost-Averaging, Value-Averaging, and Proportional Rebalancing

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Abstract

This study compares the performances of three popular strategies in the financial press, i.e., dollar-cost-averaging, value averaging, and proportional rebalancing. Monte Carlo simulations show a high frequency of dominance for the value-averaging approach in generating a higher terminal value in an individual's 401(k) retirement portfolio over the other two strategies. In addition, total risk of the portfolio is lower under value averaging than under dollar cost averaging. Value averaging provides the highest reward-risk ratio as well as highest likelihood of meeting the investment goal. Based on the overall consideration of terminal value, total risk, modified Sharpe ratio, downside risk, and dominance frequency, a targeted annual growth rate of between 10% and 12% for the equity account should be used as the target growth rate in conducting value averaging.

Jel Classification: G10; G23

Keywords: 401(k) investing, dollar-cost-averaging, value-averaging, proportional rebalancing, simulation

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1. Introduction

Three strategies are frequently discussed in the financial press for retirement investing. They are dollar-cost-averaging (DCA), value-average (VA), and proportional rebalancing (PR).[1] Under dollar-cost-averaging, investors make the same contribution to their portfolios at the same time each period regardless of whether the market is up or down. Similarly, the approach of proportional rebalancing requires that investors periodically adjust the composition of securities to their target proportion, e.g., 30% in bonds and 70% in equities - the optimal mix. Under value averaging, investors mechanically move funds into or out of the portfolio so that a dollar amount target is reached in each period. All three are mechanical trading strategies that help avoid emotional pitfalls in investing.

Addressing the fact that most participants in 401(k) plans do not have the large amount of funds required to allow them to pursue value averaging in its pure form, Chen and Estes (2007) propose a feasible value averaging approach in the 401(k) plan framework (hereafter 401(k) VA). In this approach, investors use the bond portfolio as a capital reserve and the equity portfolio as the driver for the growth of the retirement account. For each investment period, surplus funds are moved out the equity account into the bond account if the growth target in the equity account is met, or additional funds from the bond account are moved into the equity account if the growth target is not met. Chen and Estes conducted back-testing with historical data and Monte Carlo simulations to show that 401(k) VA outperforms DCA in achieving a

[1] See for example, Knight and Mandell (1993), Marshall (2000), and Leggio and Lien (2003).

higher terminal value at retirement. However, they did not compare the 401(k) VA approach to the proportional rebalancing strategy.

This study conducts simulations to compare the merits of all three strategies within the framework of a 401(k) plan. The rest of the paper is organized as follows. Section 2 discusses data and methodology. Section 3 reports simulation results. Section 4 summarizes and concludes.

2. Examples of the Three Strategies – A comparison

The approach of dollar-cost-averaging stipulates that an investor periodically makes the same contribution e.g., $1,000 with 70% in stocks and 30% in bonds, regardless of whether markets are up or down. Under the proportional rebalancing strategy, funds are moved between the bond account and the stock account so that the composition of the portfolio always conforms to a pre-determined proportion, e.g., 70% in stocks and 30% in bonds, at the end of each period.

Under 401(k) VA, the stock account is used as the driver and the bonds account as a natural reserve. If the pre-determined growth rate for the stocks account is met, money is moved out of the stock account into the bond account to build up capital reserve. In contrast, if the pre-determined growth rate for the stock account is not met, money is moved out of the bonds account into the stock account – provided that the amount of addition to the stock account does not exceed the balance of the bond account.

Notice that both strategies are similar to DCA in that they follow mechanical rules, which helps individual investors to avoid the classic emotional rollercoaster ride in investing.[2] However, different from DCA, both the 401(k) VA strategy and the proportional rebalancing strategy are more active, which may afford investors a better opportunity to "buy low and sell high".

[2] Instead of a calendar-based mechanical system, all three strategies can also be event-triggered.

Table 1 presents a comparison of the three strategies. When the equity return in the previous month is positive and the growth rate above the 1% monthly growth target under the 401(k) VA, the month's ending balance of the stock account is largest under PR and lowest under DCA. In contrast, when the equity return in the previous month is negative, the month's ending balance is highest under the 401(k) VA and is the lowest under DCA.

Of course, the results in Table 1 are stylized because the beginning balance is assumed to be the same for all three strategies. Since the monthly returns for bonds and stocks vary, it is not possible to infer from the example in Table 1 how these three strategies will fare in terms of the ending balances on a future date. To shed light on the performance of the three strategies, we conduct Monte Carlo simulations.

2. Data, Assumptions, and Methodology

2.1 Data and Assumptions

Chen and Estes (2007) based their return assumptions on historical data. For example, data from 1934 to 2003 as provided by the Federal Reserve show an average monthly return of 0.33%. They assume that stocks will generate a monthly return of 0.9%, which is the average monthly return of the Standard & Poor's 500 index from 1926 to 2003 as provided by the Center for Research in Security Prices (CRSP). In addition to extending these two historical data series to July of 2009, we also calculate the average monthly return of the stock market index provided by Global Financial Data, which extends the index back to 1900.

As in Chen and Estes (2007), we adopt a normality assumption for the return process for both bonds and stocks. Based on the extended T-bill rate data, we assume that the average monthly rate of return is 0.32% and the standard deviation is 0.26% for the bond portfolio. For comparison, Chen and Estes (2007) assume that standard deviation is 0.267%. We also downwardly adjust the average monthly stock returns to 0.6% and increase the standard deviation to 6.29%. For comparison, Chen and Estes (2007) assume that average return and standard deviation are 0.639% and 5.627%, respectively.

2.2 Methodology – Simulation Design

As in Chen and Estes (2007), we use an investment horizon of 360 months, i.e., 30 years. A horizon of 30 years is typically used in simulations, see e.g., Cooley, Hubbard, and Walz (2003), Stout and Mitchell (2006), and Schleef and Eisinger (2007).[3] We use three levels of monthly 401(k) contribution in the amount of $500, $1,000, and $1,500. For the allocation ratio as in DCA and PR, we use two, i.e., 40-60 and 30-70 for bonds and stocks. The former represents typical allocation of the traditional 40% bonds, 50% stocks and 10% short term funds and the latter is a moderately more aggressive balanced portfolio strategy.[4] For the growth rate under 401(k) VA, we choose several rates between 0.5% and 1.1%. We conduct 10,000 simulations.

Different from Chen and Estes (2007), we address the issue of what to do with the money when the retirement goal has been met midstream over the 360 periods. Chen and Estes (2007) assume that investors continue the strategy all the way to the end of the 30 years even if their investment goal has been met already. In practice, it is much more prudent to "take the money

[3] Thirty-year horizon also allows for simplicity without touching on the issue of funds withdraw from the retirement account, see e.g., Spitzer, Strieter, and Singh (2007) for a discussion on the optimal withdraw strategy.

[4] Because the main conclusion remains intact, we only report the results from the setting of an allocation of 30% for bonds and 70% for equity and a monthly contribution of $1,000.

off the table" when the investment goal has been reached. As a result, we design the simulation to allow investors to put all the funds into the bond account as soon as their investment goal is reached at the end of any month. We select $1 million as the investment goal. This amount is used because the future value would be around $1 million if one invests $1,000 per month earning an average monthly return of 0.5%, which is the weighted average of a bond return of 0.3% and an equity return of 0.6%.

2.2 Evaluation Criteria

As discussed in Chen and Estes (2007), the focus is on the terminal value of the investment under the three strategies. Therefore, we first compare the performance based on the dollar amount terminal value (TV) of the retirement account and their standard deviation. In addition, we adopt the following three evaluation criteria to compare the performances:

1. Modified Sharpe Ratio -- the ratio of the mean terminal value in excess of a minimum acceptable level over its standard deviation. Notice that the reward-risk ratio is modified from the original definition of Sharpe ratio because of the use of a dollar return instead of a rate of return as the numerator. We select $675,000 as the minimum acceptable terminal value, which is the future value of an annuity with monthly payment amount of $1,000 earning a return equal to the T-bill rate over 360 periods.

$$Modified\,SharpeRatio = \frac{(MeanTV - MinimumAccepteableTV)}{\sigma(TV_t)} \qquad (1)$$

2. Downside Risk as measured by squared root of semi-variance as in Equation 2. The semi-variance measures the average deviation of terminal value below the target level. The semi-variance is also called the 2nd-degree lower partial moment of the terminal values. Different

from standard deviation, semi-variance measures only the downside risk. We use $675,000 as the minimum acceptable terminal value.

$$Semi-variance = \frac{\sum_{1}^{m}(\max(0, MinimumAcceptableTV - TV_t))^2}{m} \quad (2)$$

3. Dominance Frequency -- how many times out of the *n* simulations that the terminal value from one strategy is higher than that from another strategy.

$$Frequency = \sum_{1}^{n} f(\Delta TV) \quad (3)$$

where $f(\Delta TV)$ takes a value of one if it is positive and zero otherwise.

3. Empirical Results

3.1 Mean Terminal Values

Table 2 presents the mean terminal values from the three strategies. Because 401(k) VA depends on the monthly growth target, simulations are conducted at each of the 7 growth targets. The mean terminal value is slightly more than $950,000 for all three strategies. DCA always outperform the proportional rebalancing strategy in generating a higher terminal value for the retirement account.

As shown in Panel B of Table 1, DCA also outperforms 401(k) VA when a monthly growth target rate below 0.7% is used in executing value averaging. However, when a monthly growth rate of 0.7% or higher is chosen, value averaging produces a higher terminal value than DCA. For example, at a monthly growth target of 1%, VA produces $27,664 more in terminal value than DCA, which is both economically and statistically significant.

At a monthly growth rate no lower than 0.7%, value averaging also outperforms PR in generating a higher terminal value of the retirement account. Thus, results in Table 1 show that the approach of proportional rebalancing is a lesser attractive option. The results also confirm that value averaging does outperform both DCA and PR in generating a higher terminal value for the retirement account.

3.2 Total Risk Profile

Figure 1 shows the total risk as measured by the standard deviation of the terminal values for three strategies. Comparing against DCA, 401(k) VA has a lower total risk at any monthly growth target that is below 1%. Comparing against PR, 401(k) VA also has a lower total risk so long the month growth rate is below 0.8%. Notice that PR always has a lower total risk than PR.

Because both VA and PR permit more of “buy low and sell high” than DCA, results in Figure 1 indicate a positive smoothing effect in reducing volatility under value averaging and proportional re-balancing. Nevertheless, for value averaging strategy, the smoothing effect depends on the monthly growth rate selected.

When comparing VA against DCA, a combining of the results in Table 1 and Figure 1 indicates that a monthly growth rate should not be higher than 1% to capture the higher terminal value and lower total risk. On the other hand, comparing VA against PR, a monthly growth rate below 0.8% should be used to allow a higher terminal value and a lower total risk.

3.3 Modified Sharpe Ratio

Figure 2 exhibits the modified Sharpe ratio, which compares the trade-off between reward and risk. Figure 2 shows that 401(k) VA always has a better reward-risk trade-off than DCA and PR. Therefore, at a monthly growth rate higher than 1%, the total risk is higher under VA than under DCA, which is shown in Figure 1. However, such an increase in total risk is more than offset by the increase in terminal value. As a result, VA still has a higher reward-to-risk ratio than DCA.

3.4 Downside Risk

Figure 3 presents a comparison based on the downside risk, which measures the semi-variance of the shortfalls below the minimum terminal value of $675,000. At a monthly growth rate of 0.5%, 401(k) VA has a lower downside risk. At a higher growth target, 401(k) VA has the highest downside risk. However, over the remaining range of monthly growth rates, the difference is only about $25,000. Nevertheless, VA has a higher downside risk than the other two strategies.

The downside risk measures the severity of bad outcomes only. That is, only those occurrences with a terminal value less than the acceptable level of $675,000 contributes the severity of bad outcomes, whereas those good occurrences are excluded. Clearly, the strategy of "buying low" would result in even greater losses if one market low is followed by more lows. Therefore, one should expect VA has a higher downside risk unless a very low monthly growth rate is used. As a result, one needs to weigh the higher downside risk against other factors.

3.5 Dominance Frequency

Figure 4 shows the frequency that a strategy generates a higher terminal value for the retirement account than the other strategy. Out of 10,000 simulated 30-year periods, there is a 55% chance that value averaging produces a higher terminal value than dollar-cost-averaging at every growth target. Proportional rebalancing also has a higher dominance frequency over DCA – at about 60%. On the other hand, value averaging dominates proportional rebalancing about 55% of time in resulting in a higher terminal value for the retirement account.

3.6 Exposure to Equity – A Discussion

As shown in Figure 3, VA has a higher downside risk than the other two strategies. Because the equity fund is used as the main driving force for achieving retirement goal, a higher monthly growth rate would result in more a tilting of contribution toward equity. How much exposure to equity does each of the three strategies have?

To answer this question, we calculate the ending ratio of equity account balance to the terminal value. Figure 5 exhibits the results. Not surprisingly, the exposure to equity is fixed around 30% for PR because the allocation is always maintained at 30% for bonds and 70% for equity at the beginning of each period. For VA, the equity exposure increases from 30% at a monthly growth rate of 0.5% to 40% at a monthly growth rate of 1.1%. However, DCA has the lowest equity exposure at an average rate of 16%.

The results in Figure 5 are not only consistent with but also caused by those in Figure 3. Recall the design of the simulation allows a transfer of equity balance into bond account when the retirement goal is reached. As a result, the equity exposure would not be reflected in Figure 5 for DCA and VA for those occurrences in which the retirement goal is reached prior to the end of the simulation. Consequently, the results in Figure 5 are reflecting those cases in which the equity account fails to drive the retirement balance to top the goal. Intuitively, equity portion would decrease relative to bond portion in these “down” situations under DCA. In contrast, the equity portion would not decrease as much as would be under VA due to moving more funds from bond account into equity account to make up the shortfall in equity. Therefore, a higher downside risk as in Figure 3 would drive up the equity exposure under value averaging more than under dollar-cost-averaging. In return, a higher equity exposure at any period could also increase downside risk if a market low is followed by another market downturn.

It is comforting to notice that there is only a 40% chance that the average equity exposure is at least 70% out of the terminal value. Results in Figure 5 complement those in Figure 1 to further indicate the positive smoothing effect from value averaging. Following the average investors reaction to the market downturn in 2008, the value of this smoothing effect needs to be appreciated because investors inevitably get it wrong and miss a high proportion of the upturn if they simply pull out of the market and wait for better times.

Given all the factors, how should we judge the merits of value averaging? As in any investing, reward comes with risk. Figure 3 shows that VA has a higher downward risk than DCA and PR. However, value averaging not only increases terminal value as shown in Table 2 but also has a lower total risk as shown in Figure 1. Based on a reward-to-risk ratio basis, Figure 2 clearly favors VA over the other two strategies. We conclude that investor should use a month growth rate that is higher than 0.7% but lower than 1% to execute value averaging to capture the benefits of 1) higher terminal value, 2) a lower total risk, 3) a reasonable downside risk, and 4) a 55% chance of dominance frequency.

4. Summary and Conclusion

This study examines the performances of the three most popular strategies in financial planning literature and press, i.e., dollar-cost-averaging, a feasible value-averaging, and proportional rebalancing. Monte Carlo simulations show that value averaging at a monthly growth rate higher than 0.7% generates a higher terminal value for the retirement account than dollar-cost-averaging or proportional rebalancing. Further, the terminal value has a lower standard deviation under value averaging than under dollar-cost-averaging when a monthly growth rate lower than 1% is used in executing value averaging.

On the other hand, value averaging does have a higher downside risk than the other two strategies. However, the equity exposure under value averaging is not as high as feared. The modified Shape ratio clearly favors value averaging in terms of a higher compensation of reward for bearing the risk. In addition, value averaging has a 55% chance of generating a higher terminal value over dollar-cost-averaging and a 52% chance over proportional re-balancing. Based on the empirical results, we conclude:

1. Both value averaging and proportional rebalancing have a higher positive smoothing effect in reducing total risk than dollar-cost-averaging.
2. Value averaging is the most preferred strategy over dollar-cost-averaging and proportional rebalancing in the framework of investing for retirement via 401(k) plans.
3. A monthly growth rate higher than 0.7% but lower than 1% for the stocks account is optimal in executing value averaging.

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Table 1. A Comparison of the Three Strategies

This table presents examples for the three strategies, i.e., dollar-cost-averaging (DCA), 401(k) value averaging (VA), and proportional rebalancing (PR). A monthly contribution of $1,000 and a target allocation is 30% for bonds and 70% for equities for all three strategies. In addition, a monthly growth target of 1% is assumed for the strategy of 401(k) VA. Given hypothetical returns for bonds and equities, adjustments are made under each strategy. Under DCA, an investor divides the monthly contribution and adds the portion to the previous balance of the bonds account and the stocks account, respectively. Under PR, an investor adds the monthly contribution to the combined balances of the bonds and the stocks accounts and divides the amount according to the target proportion. Under 401(k) VA, an investor adjusts the balance of the bonds and that of the stocks account according to whether or not the monthly growth target for the stocks account is met.

Panel A. Positive Equity Return – Previous Month's Bond Return of 0.5% and Equity Return of 3%

		DCA		401(k) VA		PR	
		Bonds	Stocks	Bonds	Stocks	Bonds	Stocks
Month k-1	Balance	$10,000	$20,000	$10,000	$20,000	$10,000	$20,000
Month k	Balance	$10,050	$20,600	$10,050	$20,600	$10,050	$20,600
Month k+1	Adjustment	+$300	+$700	+$700	+$300	30%	70%
Month k+1	Total	$10,350	$21,300	$10,750	$20,900	$9,495	$22,155

Panel B: Negative Equity Return – Previous Month's Bond Return of 0.5% and Equity Return of -10%

		DCA		401(k) VA		PR	
		Bonds	Stocks	Bonds	Stocks	Bonds	Stocks
Month k-1	Balance	$10,000	$20,000	$10,000	$20,000	$10,000	$20,000
Month k	Balance	$10,050	$18,000	$10,050	$18,000	$10,050	$18,000
Month k+1	Adjustment	+$300	+$700	-$1,900	$2,900	30%	70%
Month k+1	Total	$10,350	$18,700	$8,150	$20,900	$8,175	$20,335

Note: Under 401(k) VA, the losses in the stocks account is made up by moving funds in the bonds account into the stocks account to meet the growth target, limited by the balance of the bonds account, as is the case in Panel B.

Table 2. Monte Carlo Simulation Results –Terminal Values

For each strategy, an investment horizon of 360 months is used and monthly contribution is $1,000. For all three strategies, the allocation ratio between bonds and stocks is 30% and 70%. For 401(k) VA, different growth rates are used. In parentheses are the t-statistics testing the null hypothesis that there is no difference in the mean terminal value from each of the three strategies.

Panel A. Terminal Values

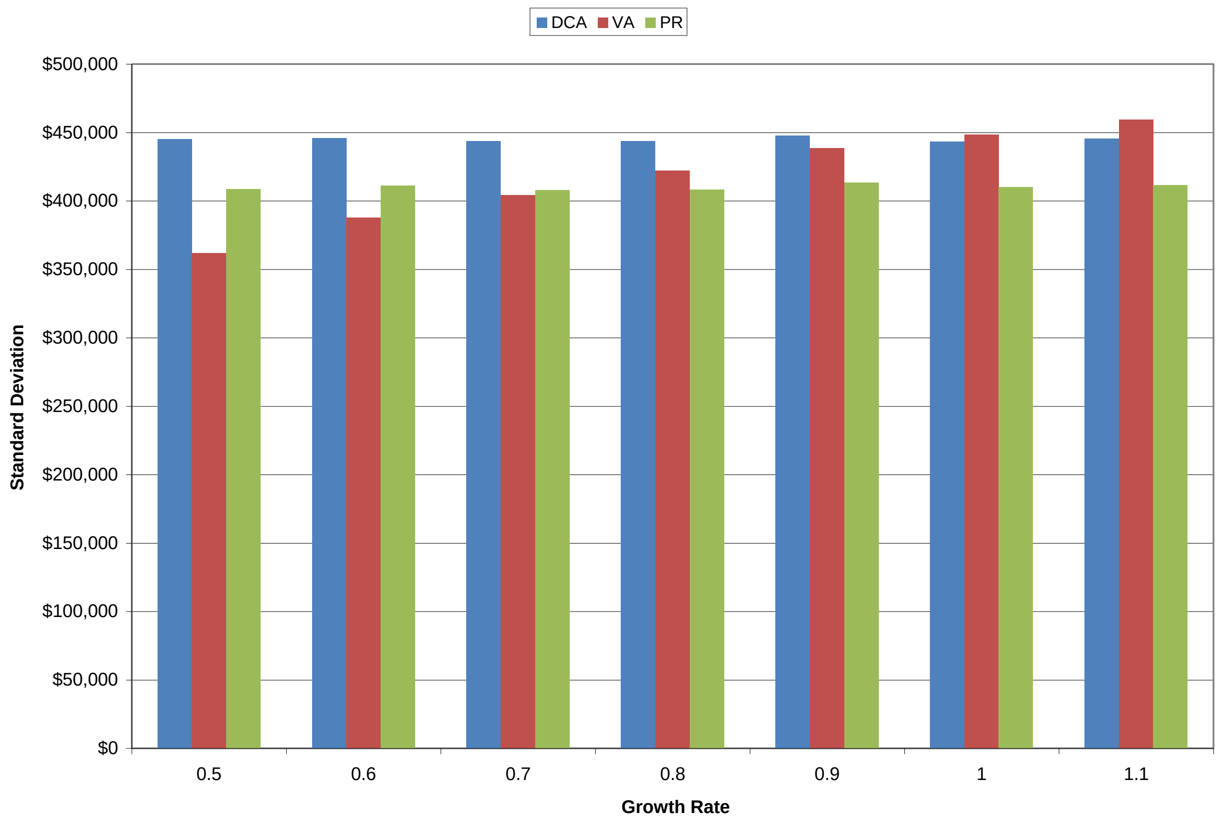
Growth Rate	Dollar-Cost-Averaging	401(k) VA	Proportional Rebalancing
0.5%	$970,730	$954,028	$965,780
0.6%	$960,484	$956,714	$956,779
0.7%	$959,197	$965,357	$955,771
0.8%	$957,496	$970,612	$952,759
0.9%	$965,150	$986,913	$962,124
1%	$964,457	$992,120	$961,459
1.1%	$960,962	$995,149	$957,236

Panel B. Difference in Terminal Values

Growth Rate	VA-DCA	VA-PR	PR-DCA
0.5%	-$16,702 (-8.10**)	-$11,772 (-8.04**)	-$4,930 (-4.47**)
0.6%	-$3,769 (-1.95*)	-$64 (-0.05)	-$3,705 (-3.48**)
0.7%	$6,161 (3.32**)	$9,586 (7.19**)	-$3,426 (-3.16**)
0.8%	$13,116 (7.38**)	$17,853 (13.20**)	-$4,737 (-4.34**)
0.9%	$21,764 (12.61**)	$24,790 (18.58**)	-$3,026 (-2.94**)
1%	$27,664 (16.68**)	$30,661 (22.56**)	-$2,998 (-2.85**)
1.1%	$34,188 (20.62**)	$37,913 (25.89**)	-$3,726 (-3.49**)

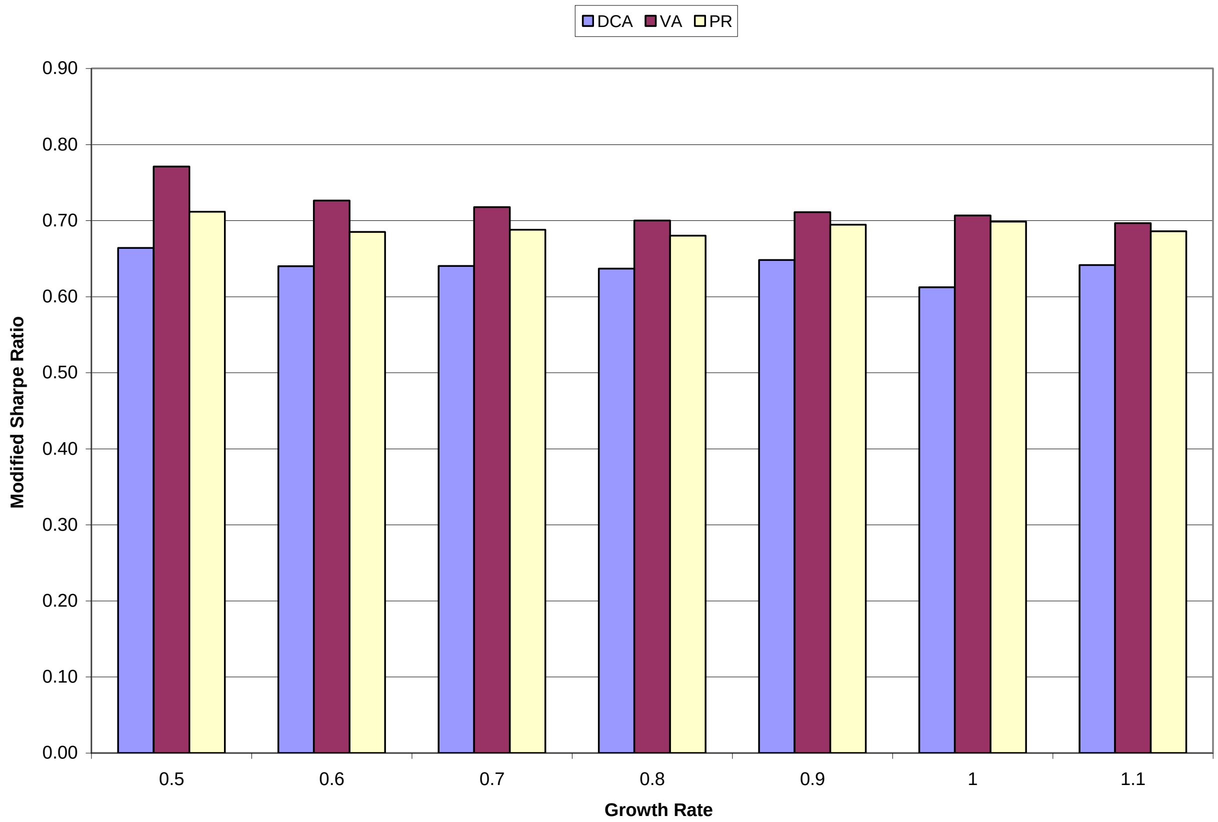
** and * indicates statistical significance at the 0.01 level and 0.05 level, respectively.

Figure 1. A Comparison Based on Total Risk



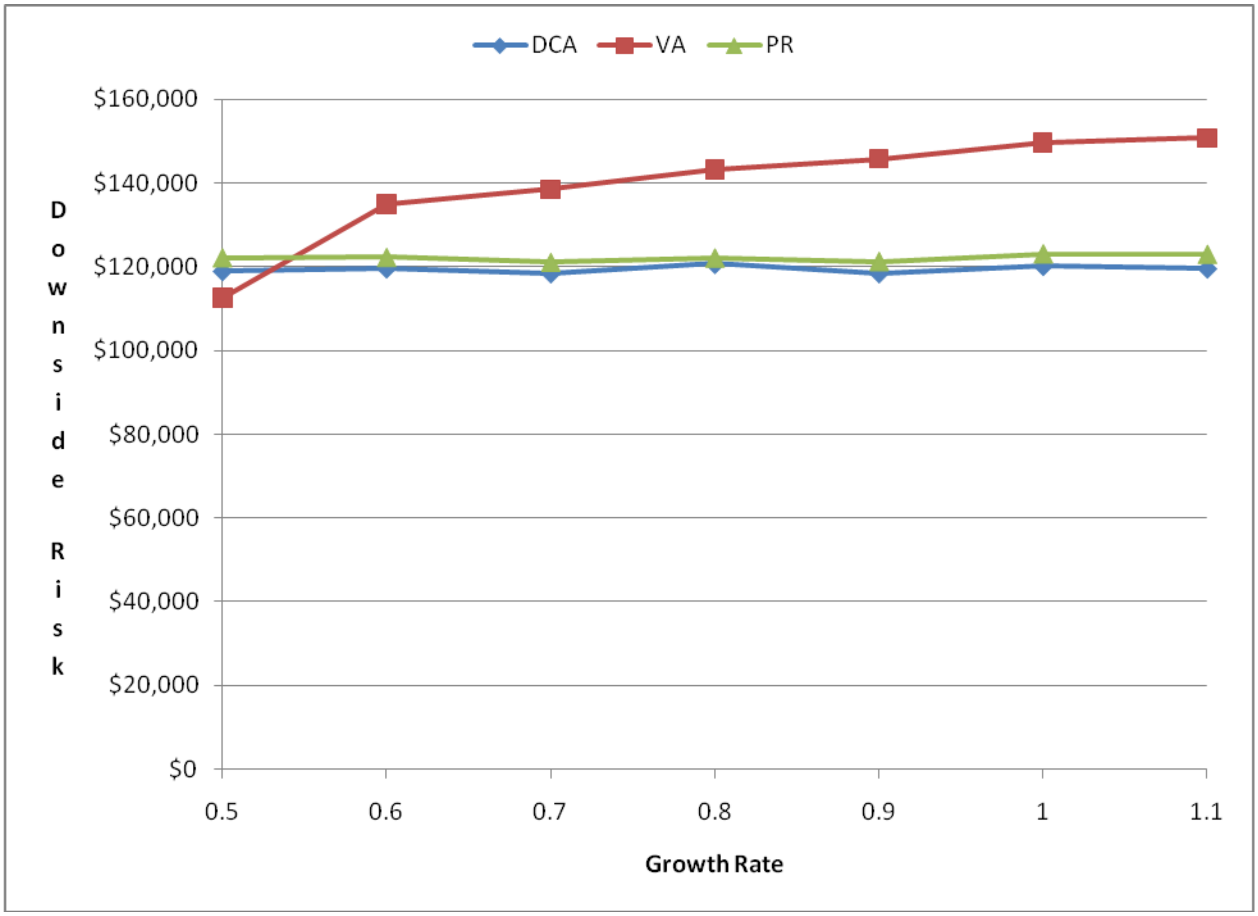
Note: Total risk is measured by the standard deviation of the terminal value of the retirement account.

Figure 2. A Comparison Based on Modified Sharpe Ratio



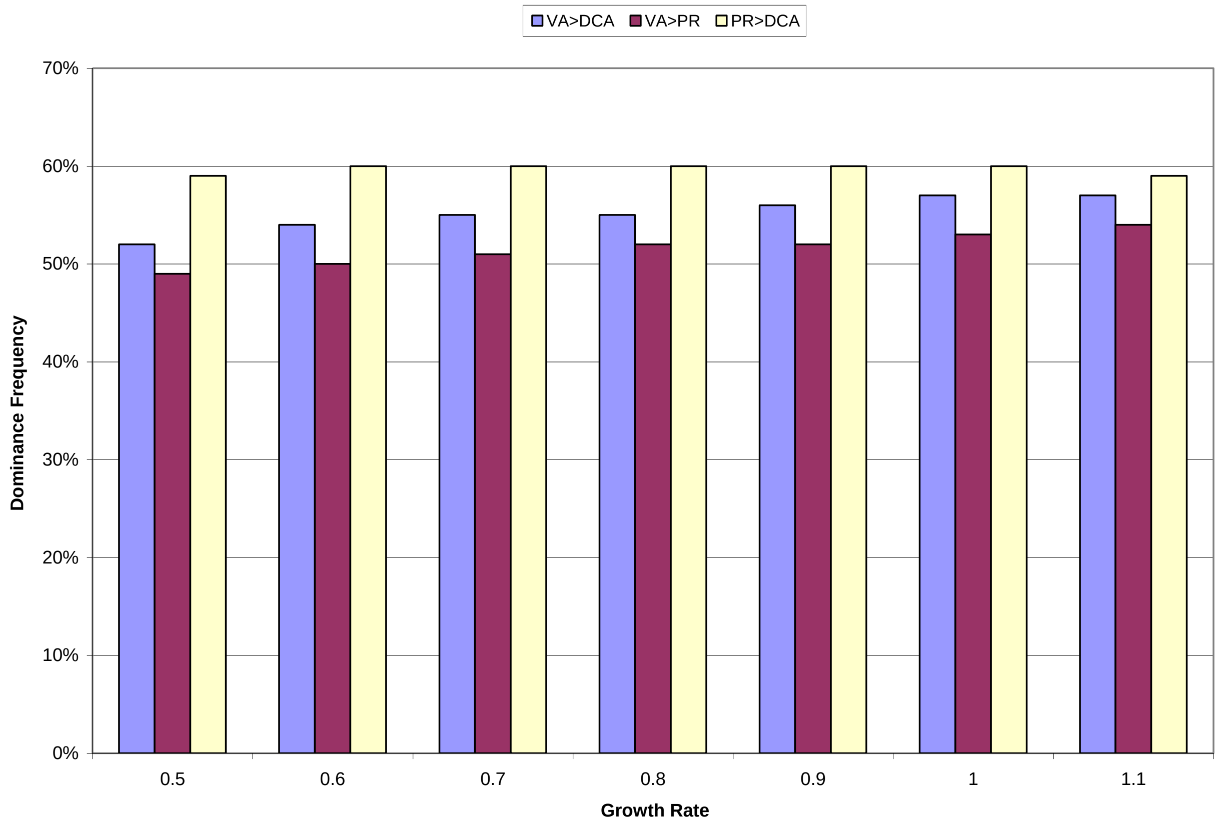
Note: Modified Sharpe ratio is calculated by on the minimum acceptable terminal value of $675,000, which is the future value of an annuity with monthly payment amount of $1,000 earning a return equal to the T-bill rate over 360 periods. A higher ratio indicates a best risk-reward tradeoff.

Figure 3. A Comparison based on Downside Risk



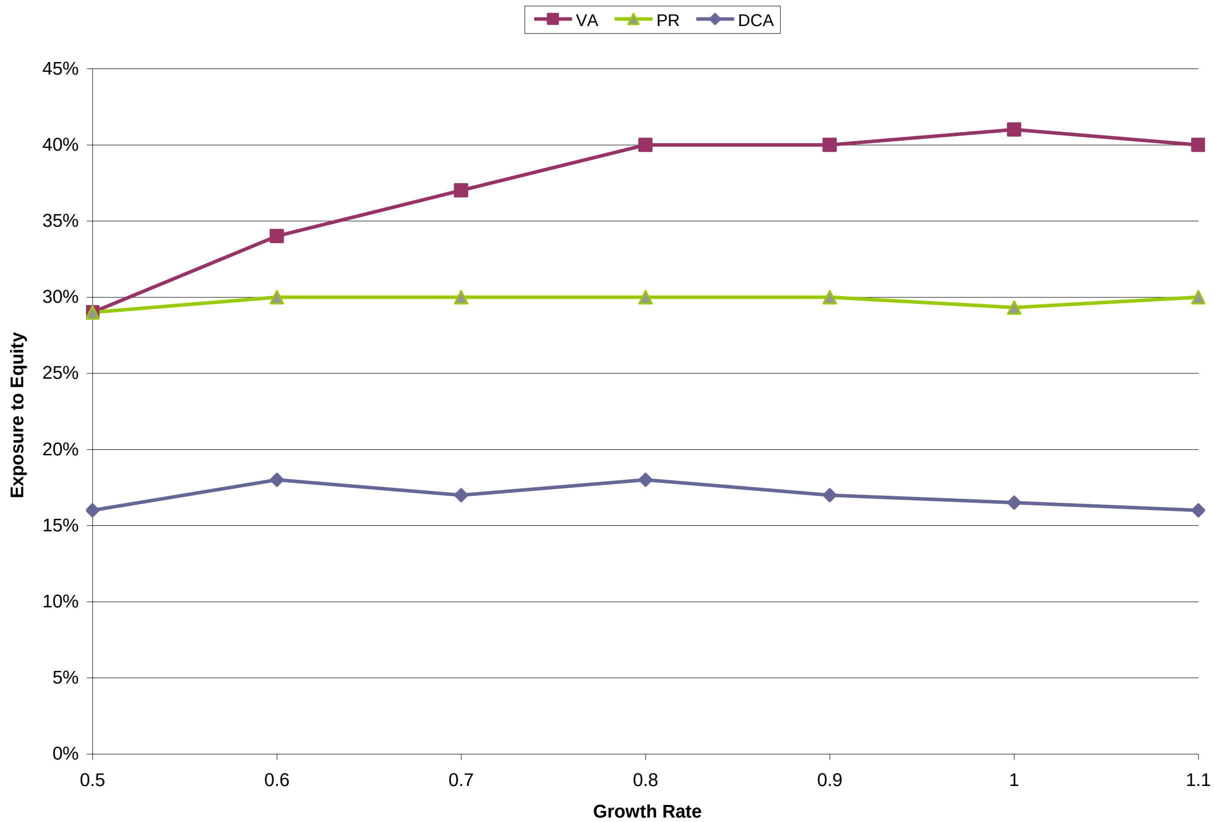
Note: Downside risk is the semi-variance of the terminal values that are below the target level of $675,000, which is the future value of an annuity with monthly payment amount of $1,000 earning a return equal to the T-bill rate over 360 periods. It measures the average dispersion of terminal value below the acceptable level.

Figure 4. A Comparison Based on Dominance Frequency



Note: Dominance frequency measures the occurrences out of 10,000 simulations such that the terminal value from one strategy is larger than that from another strategy.

Figure 5. Degree of Exposure to Equity



Note: This figure exhibits the frequency that the ratio of equity account balance is at least 70% of the terminal value of the retirement account. It measures the degree of exposure to equity under each of the three strategies.