

# **A multi-market, historical comparison of the investment returns of value averaging, dollar cost averaging and random investment techniques**

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## ABSTRACT

As the title suggests, this paper compares two "formula" or mechanical investment techniques, value averaging (VA) and dollar cost averaging, to a form of random investing to determine if any technique yields superior investment return performance. The tests use historical market prices of chosen stock and commodity indices. Results seem to indicate that value averaging does provide a small but still superior expected investment returns under most conditions. Due to the relatively few real world "experiences" available, these results can only be anecdotally and not statistically confirmed at a high confidence level. Actual investment results reported here are consistent with prior statistically significant research supporting a small investment performance advantage for value averaging versus both other techniques using simulation to approximate market activity. Evidence builds that VA works!

## INTRODUCTION

An earlier paper (Marshall and Baldwin, 1994) did a statistical comparison of simulation based investment results for Dollar-Cost Averaging (DCA) and random investment techniques. They calculated the internal rate of return (IRR) to an investor from each of many simulated investment scenarios under both techniques. Their research question was, "Does DCA yield superior investment performance compared to a purely random investment technique?" They found, with 99% confidence, that there is no statistical difference in the IRRs achieved by each technique. They also found, with 95% confidence that each technique had the same risk as measured by the standard deviation of the IRR distributions. They concluded that the null hypothesis was valid and that DCA was not superior to random investments. These results are contrary to most practitioner given investment advice, even including Vanguard's (Vanguard, 1988), and contrary to that presented in many texts on personal finance. See for example (Gitman and Joehnk, 2002.)

Even with those rather astounding results, recent discussion of VA has been sparse, save for a short favorable mention in the Wall Street Journal (Clements, 2001). Amazingly, no other published academic research other than Edleson and Marshall's has tested Value Averaging. Even the popular press is almost silent on VA, particularly when compared to continuing discussion of the now fully academically discredited DCA.

Why such silence? Who knows? Hopefully this research may help to correct that deficiency by continuing the debate by testing the investment performance of VA against both DCA and random investment techniques in the real world of actual market prices. Instead of a theoretical

or simulation based approach, this paper proposes an empirical test of the investment performance of DCA, VA and random investing on actual market data over extended (and variable) investment time horizons. Furthermore tests will include foreign as well as domestic markets and other than equity markets, as suggested by some (Bacon, 1997). The research question employed in this paper is,

"Is there evidence that VA yields superior investment return performance compared to DCA or to a purely random investment technique when tested on actual market data across multiple markets and variable investment time horizons?"

#### A DESCRIPTION OF TECHNIQUES: DOLLAR COST AVERAGING, VALUE AVERAGING AND RANDOM INVESTING

Instead of asking the reader to review other work as a primer on both DCA and VA, perhaps that chore can best be accomplished here? Also, the exact definition used for Random investing needs description. DCA is generally well understood. Perhaps Yahoo's glossary (Yahoo, 2004) definition for "constant dollar plan" (as they call DCA) is as good as any:

"(DCA is ...) a method of purchasing securities by investing a fixed amount of money at set intervals. The investor buys more shares when the price is low and fewer shares when the price is high, thus reducing the overall costs."

It is the essence of a buy and hold strategy. There is no talk of selling. Similarly, there is no suggestion as to how long DCA should be applied. Their choice of language is also interesting and biased. Can there be any doubt among average investors that, "... reducing overall costs," and by extension, DCA, is a good thing?

The inventor (Edleson, 1991) of Value Averaging, believes the idea behind it is simple. The investor sets a predetermined value or worth for his portfolio in each future time period, as a function of the size of the initial investment, the size of periodic investments and the investment return expected. The investor then buys or sells sufficient "shares" or units of the investment such that the predetermined portfolio worth is achieved at each revaluation point. On yield expectation, the author (Edleson, 1991, p. 119) suggests a long run equity return of 16% (which now seems absurdly high in this post-NASDAQ bubble world), based on an equity return 7.4% higher than the then existing rate on long term bonds. On revaluation timing, the author (Edleson, 1991, p. 162] suggests that, "... (using) value averaging two, three or four times a year would be reasonable..." In his own words, the author (Edleson 1988, p. 13) defines the value-averaging concept:

"The rule under value averaging is simple: ... make the value not (the market price) of your stock go up by a fixed amount each month."

Considering movements in the investment's market price, the investor then either acquires or disposes of sufficient units of the investment such that the investment's required value is achieved at each subsequent revaluation point. During periods of market price decline, the investor is required to purchase relatively many units to maintain portfolio value. Conversely,

during rising markets the technique requires the purchase of relatively few shares to achieve required value. During extended bull markets or during unusually large upward spikes in market price, the technique requires that units be sold to maintain portfolio value at the desired level.

The VA technique is even more intuitively appealing than DCA. As with DCA, more investment units are purchased when prices are low. However, VA magnifies the need to purchase relative to DCA since unit price declines reduce the value of the portfolio thus increasing the need for extra investment and initiating ever more aggressive "buy" signals. Furthermore, and contrary to DCA, VA gives a rule for selling. As the market price increases, beyond what it was recently, VA may require unit sales since the growing price rise may substantially increase the value of the portfolio. And, if the market price continues to increase dramatically, VA gives ever more aggressive "sell" signals to control the value of the portfolio to the level desired.

In the earlier work (Marshall and Baldwin, 1994, p. 61) it is stated that DCA was appealing because,

"Intuitively, DCA is contrary in the sense that fewer shares are purchased when price are 'high' and more shares are purchased when price are 'low', facilitating the 'buy low' aspect of the ancient investment adage, 'buy low, sell high'."

VA conceptually does an even better job. Even more units are purchased at "low" prices and probably some, at least, are sold at "high" prices.

At this stage, a numerical description of VA and a comparison to DCA may be useful. The price pattern in Table 1 shows that whether the market price of an investment is rising, falling, or fluctuating over time, VA yields a lower average cost of shares purchased than does DCA and both are lower than the average price of shares. No proof, nor even contention, is offered here that this happens under all price patterns, but the specific price patterns used are not selected solely to achieve this goal. The price patterns are the same ones used by Vanguard to tout the supposed benefits of DCA, and the same ones used by Marshall and Baldwin and Marshall in their research.

The mathematical "certainty" (as reported by others, see (Edleson, 1991, p. 30) that DCA average cost is always lower than the average price has allowed some to promote DCA as an attractive way to assure superior investment performance. If that were sufficient to assure superior investment performance then by definition VA must be a superior to DCA since VA's average cost is lower than DCA's. But, as demonstrated by Marshall and Baldwin, if there is no statistical difference in investment returns as measured by IRR between DCA and random investing, then logically, random investing must on average acquire shares at the same cost as DCA, time and value considered. Therefore, by extension, the fact that VA acquires shares at lower average cost than DCA for these examples, or even in all cases, is not enough to assure that VA has a performance advantage over DCA. Statistical tests are necessary, and possible due to the essentially unlimited "testing" potential of simulation.

The IRRs for both VA and DCA are shown in Table 1. Interestingly, but not necessarily statistically significant, VA has a higher IRR than DCA for each market price pattern shown. To

calculate each technique's cash flow pattern, the length of the investment time horizon, the dollar amount invested and the market price of the investment in each period are required. The IRR can then be calculated since the amount and timing of each periodic investment (or disinvestment) and the ending market value of the portfolio are known. For example, in a rising market as shown in Table 1, the "Period Invest" column for DCA requires a cash outflow of \$400 each period, 1 through 4. After a final investment of \$400 in the fifth period, the DCA investor has acquired 235 shares with a market price of \$16 a share for a total portfolio value of \$3,760. The IRR of the cash flow is 32.01%, assuming annual time periods and no transaction costs or taxes.

Some may argue that Table 1 is flawed. The "Value Required" column of VA is simply equal to the cumulative investment shown under the "Total Invest" Column of DCA, implying that the VA investor expects no return on investment. To counter that argument, to better match Edleson's methodology, and to further demonstrate the VA investment technique, Table 2 is presented. Table 2 allows the "Value Required" column of VA to increase period to period by 10% of the prior period's "Value Required" plus the same \$400 "Period Invest" shown for DCA, thus implying a 10% investment growth per period for VA. Again, the results are similar to Table 1. Each test shows VA with a lower average cost of shares than DCA and higher IRRs. However, the important question is not which technique yields the lower average cost of an investment. What really matters is which technique yields the statistically significant best investment performance.

This paper uses the same definition of "random" as in prior work. Random investing includes a 50% probability of investing in a particular period and a 50% probability of sitting idle. When an investment is made there is an equal chance of investing either 150% or 250% of the amount invested each period with DCA. This procedure carries three advantages. First, it probably better approximates normal investment pattern such as "on / off" or "more / less" common among many investors, particularly outside of 401K type retirement plans. Second, the probabilities assumed in the technique guarantee that the expected value of the investment is the same as in DCA. This prevents a potential bias in the comparisons by investing considerably more in one technique than the other. Third, it duplicates the method followed in prior work, thus making comparison to that work easier.

## EXPERIMENTAL METHODOLOGY

This paper closely follows earlier methodology, (Marshall, 2000 and Marshall and Baldwin, 1994) and uses the same three-way analysis proposal (VA vs. DCA and random investing.) The method used in this paper to calculate the return associated with each investment technique is simply to calculate the IRR of the cash flow that results from employing the technique being evaluated over the investment time horizon chosen, then cashing-in the investment value at the end of the time horizon, just as shown in Tables 1 and 2. For both DCA and random techniques no money is returned to the investor except at the end of the time period. For VA money may be returned at any time the technique gives a partial "sell" signal. Each technique's return is determined by the procedure described and the actual performance of the underlying market in the particular time period under analysis.

IRRs are calculated for consecutive five and 10-year investment time horizons for each market studied as well as for the entire length of data utilized in each market. A five-year investment time horizon is suggested to be appropriate by many investment writers, (Gitman and Joehnk, 2002.) A ten-year (and for S&P 500 only a 20-year) time horizon is used when the effect of investment time horizon is tested to help prove or disprove VA's superior performance. The logic seems to be that any system that could improve investment returns would be favored over longer time horizons where VA had time to work its "magic" and its benefits would compound. Sometimes data is broken out for "up" markets and "down" and results are sometimes offered weighted by the number of years of data available in each index.

The multiple markets studied are represented by the following indices: the S&P 500 from 1871 to 2002 (Schiller, 2003) and from (Yahoo, 2003) the Dow Jones Industrial Average (1932-2002), the FTSE 100 (1984-2002), the Philadelphia Exchange Gold and Silver Index, the XAU (1983-2002) and the Dow Jones Commodity Index (1980 -2002.) Those market indices were chosen for the following reasons:

- \* The S&P 500 is generally recognized as representative of broad U.S. equity values. Schiller's data, is available free on-line and is perhaps the longest stock index consistently calculated save for one series (Siegel, 2002), with data going back to 1802.--The Dow just had to be included as the most popular stock market average, poorly constructed, as it may be to finance professionals. If VA "works" it must do so for badly constructed indices as well.

- \* The FTSE was included to reflect the performance of foreign stocks. If VA "works" it should not care which side of the Atlantic it is on.

- \* The Dow Jones Commodity Index and the Philadelphia Exchange Gold and Silver Index were included in the study for two reasons. First neither relates to equity markets. Clearly, if VA "works" it should work in all markets. How would it know whether the price changes input to it were stock prices, gold prices, or bananas prices at the local market? Secondly, neither gold nor commodities more generally have performed well over the period provided by YAHOO. Both have experienced slightly negative (about negative 1%) annual returns. If VA "works" it should do so long term in both increasing and decreasing markets.

Edleson's methodology requires that the VA technique employ an expected return assumption that along with the assumed periodic investment determines the required investment value at the end of each revaluation period. As mentioned earlier, Edleson [4] in 1991 thought 16% appropriate. Conversely, Schiller's work (Schiller, 2003) creating a very long term S&P 500 Index with dividends reinvested found 6.9% to be the average return. That return is used here. No initial investment was assumed for any analysis and equal, or the expectation of equal, periodic investments are used. A quarterly revaluation period for VA and a quarterly investment period for both DCA and random investing are used, consistent with prior work.

## RESULTS OF COMPARABLE INVESTMENT ANALYSIS

Table 3, although totally unsophisticated statistically, is interesting. It shows how many times each technique placed first, second or third as measured by IRR, when applied to rolling 5-, 10-,

20-year periods of S&P 500 index data. This is the only application of the 20-year investment horizon. In other markets too few periods were available to be meaningful.

The entire S&P index running for more than 130 years was also tested. The highest IRR among the three investment techniques determines first place; the lowest, third place, with no regard as to size of the margin of victory or loss. Four results attract the eye!

\* VA combined results appear to dominate DCA. VA scored 73% of all first place results, 11% of second place and 16% of third place, vs. 23%, 50% and 27% respectively for DCA.

\* DCA combined results similarly appear to dominate random investing, which placed 5%, 39% and 57% respectively. Some numbers do not add due to rounding.

\* VA relative performance increases as the investment time horizon moves from 5 years (54% first place finishes) to 10 years (92%) to 20 years (100%).

\* Less dramatically, DCA's total dominance of random investing at the 5-year time horizon becomes much less dominant at 10 and 20-year time horizons.

Marshall (2000, 93) prepared a similar chart to Table 3 showing the number of times each investment technique was superior, i.e., a first place finish, for his 6,500 simulations. Interestingly, he found that VA finished first 74% (really 73.5%) of the time vs. 73% (really 72.7%) in this paper, using actual S&P 500 investment results. An amazing coincidence? Similarly, performance improved in both papers as the time horizon increased. Strangely, the same earlier chart showed random investing dominating DCA performance; opposite to results shown in Table 3. Recall though, earlier work showed no difference in risk or return for DCA vs. random investing when sophisticated statistical analysis was applied. Also suggesting the same may reoccur, DCA fails to improve (or even maintain) performance vs. random as the time horizon lengthens.

Table 4 presents mean IRRs (%) for all tests for VA, DCA, and random investment techniques for each market as a function of the investment time horizon. Again, four results attract the eye!

\* VA produces the highest average mean IRRs for all markets--US stocks, foreign stocks gold and commodities.

\* VA produces the highest mean IRR for all time periods except for the 5-year investment time horizons for the XAU, where Random was best, and for the 5-year investment time horizon for the S&P 500 where DCA was best. In the shorter run and particularly with random events, "anything" can happen.

\* But, the absolute value of differences in investment returns among the techniques is generally small, usually on the order of 1% per year. Of course, over time an extra 1% return can be important. For example, increasing Schiller's long-run return of 6.9% to 7.9% for constant dollar annuity savings over a 35-year time horizon yields about a 25% larger nest egg.

\* Even though DCA scored a higher IRR many more times than did Random investing (see comments on Table 3) Random's average IRR actual slightly exceeded DCA's, calling any contention of DCA superiority into question.

Table 5 presents the average of mean IRRs (%) for VA, DCA, and random investment techniques, weighted and un-weighted, in up markets and down as a function of the investment time horizon. And, in the final column VA's advantage (%) vs. the average of DCA and random investing. The Table shows the "average of the means" both un-weighted and weighted by the number of years of price data used for each index, and calculated for "up" indices (S&P 500, Dow and FTSE indices) and "down" indices (DJ Commodities and XAU indices) separately. Four results attract the eye!

\* VA produces the highest combined mean IRR (i.e., the average of mean IRRs, and for both "Up" and "Down" markets, on both an un-weighted and weighted basis--everything!)

\* VA produces the highest mean IRR for all time periods except for the 5-year investment time horizons for un-weighted "Down" markets, where Random was best. Of course, as mentioned earlier, in the short run and particularly with random events, "anything" can happen.

\* It appears from looking at the combined mean IRR, for both weighted and un-weighted means, (from the last column to the right) that VA's dominance increases as the investment time horizon increases. For example, on a weighted basis VA's mean IRR advantage grows from 0.50% at 5-years to 0.71% at 10-years to 1.13% when entire periods are tested. This result is expected if VA were truly better.

\* But, the same results discussed above, when broken out to show "Up" and "Down" markets, indicate an even stronger VA advantage as time increases in "Down" markets and no recognizable similar pattern for "Up" markets. Is that result meaningful? Future research may decide.

Different from the earlier work (Marshall and Baldwin, 1994 and Marshall, 2000) this paper does not report risk statistics. The reason is not that risk should not be considered; the reason is that creating enough tests to confirm risk differences statistically requires the creation of many, many actual investment scenarios to test--and each probably should be of sufficient length in time to be meaningful to real world investors. How many? Thousands were necessary to reach some statistically significant conclusions in earlier simulation based work. While that challenge is theoretically possible it clearly goes beyond the scope of this paper.

The purpose of this paper was simply to see if there is evidence that VA has superior investment returns in the real world, not just in a simulated one. The suggestion here is to compare the results here of tests of actual investment results using real market data to simulation based results from prior work. If results are similar, then perhaps conclusions drawn can also be similar and useful?

## CONCLUSIONS

What were those results and how do they compare to earlier work? First, VA provides consistently higher returns just as it did in earlier work! But, VA's advantage over other techniques, which ranged here from about 1/2 % or less for 5-year time horizons to about 3/4 % or more for 10-year time horizons is substantially less than the approximate 1--3/4 % advantage calculated earlier (Marshall, 2000). Perhaps given more data, VA's advantage might grow toward the theoretical simulation based return difference levels, particularly in times of high price volatility? Perhaps more work along other lines proposed (Fisher, 2003) might prove useful? Second, just as in prior work, there is no indication that DCA provides any benefit to Random investing.

This paper, using actual prices achieved in multiple markets indicates that the amount of extra return associated with VA appears to be small but still interesting and potentially important to both investors and the financial services industry. Particularly in this era of low risk free interest rates, investors could clearly use an extra 1/2 % or more. The financial services industry would also benefit from a technique offering a research based "sell" as well as their plentiful "buy" signals. Prior work based on simulation indicated that VA achieved a statistically meaningful advantage in highly volatile markets and over extended investment time horizons. It is not possible in this research to statistically confirm those results using actual investment price data. Let future research, if it be deemed worthwhile, be designed specifically to further address this statistical confirmation issue. Is VA really better than DCA or Random investing? Results based on actual investment opportunities are less convincing than simulation results, but the answer appears to be, "Yes!"

## SUGGESTIONS FOR FURTHER RESEARCH

The issue of whether or not VA really provides superior investment performance in actual markets is still an open question, not totally resolved by this research though progress has been made and results seem useful if not conclusive. Earlier simulation-based research theoretically found evidence supporting the contention of VA's superiority in volatile markets and over extended time horizons. While results reported in this work do not contradict prior research, perhaps future work could be designed to focus on these issues?

Also, Edleson's description of VA requires an assumption of the yield expected on the investment portfolio. Recall, he proposed 16% and this paper used 6.9%. Does that assumption influence results? And, results reported in this paper seem to indicate that VA performs better in "Up" markets than in "Down" ones. Are those results meaningful? Future research may wish to investigate.

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Table 1: Average Prices, Average Costs and IRRs for VA and DCA in Rising, Declining, and Fluctuating Markets.

Rising Market

Value Averaging					
Period	Market Price	Value Required	Shares Owned	Shares Bought	Period Invest
1	\$5	\$400	80	80	\$400
2	8	800	100	20	160
3	10	1200	120	20	200
4	10	1600	160	40	400
5	\$16	\$2000	125	(35)	(560)
AVG	\$9.80				\$600

Average Cost (1): \$4.80  
 IRR: 33.83%

Dollar Cost Averaging

Period	Market Price	Period Invest	Shares Bought	Shares Owned
1	\$5	\$400	80	80
2	8	400	50	130
3	10	400	40	170
4	10	400	40	210
5	\$16	\$400	25	235
AVG	\$9.80	\$2000		

Average Cost: \$8.51  
 IRR: 32.01%

Declining Market

Value Averaging					
Period	Market Price	Value Required	Shares Owned	Shares Bought	Period Invest
1	\$16	\$400	25	25	\$400
2	10	800	80	55	550
3	8	1200	150	70	560
4	8	1600	200	50	400
5	\$5	\$2000	400	200	\$1000
AVG	\$9.40				\$2910

Average Cost: \$7.28  
 IRR: 24.08%

Dollar Cost Averaging

Period	Market Price	Period Invest	Shares Bought	Shares Owned
1	\$16	\$400	25	25
2	10	400	40	65
3	8	400	50	115
4	8	400	50	165
5	\$5	\$400	80	245
AVG	\$9.40	\$2000		

Average Cost: \$8.16  
IRR: 24.80%

Fluctuating Market

Value Averaging

Period	Market Price	Value Required	Shares Owned	Shares Bought	Period Invest
1	\$10	\$400	40	40	\$400
2	8	800	100	60	480
3	5	1200	240	140	700
4	8	1600	200	(40)	(320)
5	\$10	\$2000	200	0	\$0
AVG	\$8.20				\$1260

Average Cost: \$6.30  
IRR: 15.22%

Dollar Cost Averaging

Period	Market Price	Period Invest	Shares Bought	Shares Owned
1	\$10	\$400	40	40
2	8	400	50	90
3	5	400	80	170
4	8	400	50	220
5	\$10	\$400	40	260
AVG	\$8.20	\$2000		

Average Cost: \$7.63  
IRR: 13.15%

(1) Average cost can be calculated from the total of the "Period Invest" column divided by the number of shares owned at period 5. For example, using the Rising Market scenario, \$600 total investment for VA bought 125 shares for an average cost of \$4.80 a share, and \$2000 total investment for DCA bought 235 shares for an average cost of \$8.51.

Table 2: Average Prices, Average Costs and IRRs for VA and DCA Rising, Declining, and Fluctuating Markets Assuming a 10% Return for Value Averaging.

Rising Market

Value Averaging

Period	Market Price	Value Required	Shares Owned	Shares Bought	Period Invest
1	\$5	\$400.0	80.0	80.0	\$400.0
2	8	840.0	105.0	25.0	200.0
3	10	1324.0	132.4	27.4	274.0
4	10	1856.4	185.6	53.2	532.4
5	\$16	\$2442.0	152.6	(33.0)	\$(527.0)
AVG	\$9.80				\$878.8

Average Cost: \$5.76  
IRR: 33.89%

Dollar Cost Averaging

Period	Market Price	Period Invest	Shares Bought	Shares Owned
1	\$5	\$400	80	80
2	8	400	50	130
3	10	400	40	170
4	10	400	40	210
5	\$16	\$400	25	235
AVG	\$9.80	\$2000		

Average Cost: \$8.51  
IRR: 32.01%

Declining Market

Value Averaging

Period	Market Price	Value Required	Shares Owned	Shares Bought	Period Invest
1	\$16	\$400.0	25.0	25.0	\$400
2	10	840.0	84.0	59.0	590.0
3	8	1324.0	165.5	81.5	652.0
4	8	1856.4	232.1	66.6	532.8
5	\$5	\$2442.0	488.4	256.3	\$1281.5
AVG	\$9.40				\$3456.3

Average Cost: \$7.08  
IRR: -24.42%

Dollar Cost Averaging

Period	Market Price	Period Invest	Shares Bought	Shares Owned
1	\$16	\$400	25	25
2	10	400	40	65
3	8	400	50	115

4	8	400	50	165
5	\$5	\$400	80	245
AVG	\$9.40	\$2000		

Average Cost: \$8.16  
IRR: -24.80%

#### Fluctuating Market

##### Value Averaging

Period	Market Price	Value Required	Shares Owned	Shares Bought	Period Invest
1	\$10	\$400	40.0	40.0	\$400.0
2	8	840.0	105.0	65.0	520.0
3	5	1324.0	264.8	159.8	799.0
4	8	1856.4	232.1	(32.7)	(261.6)
5	\$10	\$2442.0	244.2	12.1	\$121.0
AVG	\$8.20				\$1587.4

Average Cost: \$6.30  
IRR: 15.22%

##### Dollar Cost Averaging

Period	Market Price	Period Invest	Shares Bought	Shares Owned
1	\$10	\$400	40	40
2	8	400	50	90
3	5	400	80	170
4	8	400	50	220
5	\$10	\$400	40	260
AVG	\$8.20	\$2000		

Average Cost: \$7.69  
IRR: 13.15%

Table 3: A Comparison of the Rankings of Each Investment Technique for the S&P 500 Index as a Function of Investment Time Horizon. \*

	Value Averaging	Dollar Cost Averaging	Random Investing
Entire period			
1st Place	1 (100.0%)	0	0
2nd Place	0	1	0
3rd Place	0	0	1
20-Year Periods			
1st Place	6 (100.0%)	0	0
2nd Place	0	4	2
3rd Place	0	2	4
10-Year Periods			

1st Place	12 (92.3%)	0	1
2nd Place	1	8	4
3rd Place	0	5	8
5-Year Periods			
1st Place	13 (54.2%)	10	1
	4	9	11
	7	5	12
Combined Results			
Percentage 1st Place Finishes	72.7%	22.7%	4.6%
Percentage 2nd Place Finishes	11.4%	50.0%	38.6%
Percentage 3rd Place Finishes	15.9%	27.3%	56.8%

\* A technique has a first place finish if it earns the highest IRR, irrespective of the margin of "victory." Definitions for second and third place finishes are obvious

Table 4: Mean IRRs (%) for VA, DCA, and Random Investment Techniques for Each Market as a Function of Investment Time Horizon.

	Value Averaging	Dollar Cost Averaging	Random Investing
S&P 500			
Entire Period *	9.44	9.28	9.27
10-Year	8.86	8.61	8.33
5-Years	8.89	9.16# **	7.61
Avg. All Periods	9.07	9.02	8.40
Dow Jones			
Entire Period	8.11	6.87	6.87 ***
10-Year	6.89	5.87	5.76
5-Years	7.01	5.92	6.20
Avg. All Periods	7.34	6.22	6.28
FTSE			
Entire Period	4.49	3.87	3.96
10-Year	4.34	4.18	4.29
5-Years	6.24	4.75	5.49
Avg. All Periods	5.02	4.27	4.58
XAU: Gold			
Entire Period	0.53	-1.11	-1.50
10-Year	-1.89	-3.20	-3.84
5-Years	-2.49	-2.09	0.20#
Avg. All Periods	-1.28	-2.13	-1.71
Dow Jones Commodities			
Entire Period	-0.10	-1.17	-1.05
10-Year	0.08	-1.13	-0.99
5-Years	0.87	0.42	0.72
Avg. All Periods	0.25	-0.91	-0.44
Combined Results Avg. All Periods	4.08	3.29	3.42

\* "Entire Period", by definition, has only one IRR.

\*\* *Italicized and Underlined* entries are tests where VA did not have the highest IRR. There were only two such occurrences.

\*\*\* Winner at the next decimal point.

Note: *Italicized and Underlined* entries are tests where VA did not have the highest IRR is indicated with #.

Table 5: Average of Mean IRRs (%) for VA, DCA and Random Investment Techniques, Weighted and Un-weighted, in Up Markets and Down as a function of Investment Time Horizon.

	Value Averaging	Dollar Cost Averaging
Average of Means (Un-weighted)		
Entire Period	4.49	3.54
10-Year	3.66	2.87
5-Years	4.10	3.63
"Up" Market * Only		
Entire Period	7.35	6.67
10-Year	6.70	6.22
5-Years	7.38	6.61
"Down" Market Only		
Entire Period	0.22	-1.14
10-Year	-0.91	-2.16
5-Years	-0.81	-0.84
Average of Means Weighted **		
Entire Period	7.73	6.65
10-Year	6.44	5.83
5-Years	6.65	6.38
"Up" Market Only		
Entire Period	8.59	8.03
10-Year	7.84	7.36
5-Years	8.06	7.75
"Down" Market Only		
Entire Period	0.10	-1.14
10-Year	-0.84	-2.09
5-Years	-0.69	-0.75

	Random Investing	VA Less Avg of DCA Random
Average of Means (Un-weighted)		
Entire Period	3.51	0.97
10-Year	2.71	0.87
5-Years	4.04	0.26
"Up" Market * Only		
Entire Period	6.70	0.67
10-Year	6.13	0.52
5-Years	6.43	0.86
"Down" Market Only		
Entire Period	-1.28	1.43
10-Year	-2.42	1.38
5-Years	0.64	-0.43
Average of Means Weighted **		
Entire Period	6.55	1.13
10-Year	5.63	0.71
5-Years	5.93	0.50
"Up" Market Only		
Entire Period	8.05	0.55
10-Year	7.17	0.79
5-Years	6.98	0.74
"Down" Market Only		
Entire Period	-1.26	1.30
10-Year	-2.32	1.36
5-Years	0.48	-0.42

\* "Up" markets include the S&P 500, the Dow and the FTSE; "Down" markets include the XAU and the Dow Jones Commodities indices.

\*\* Weighted by the number of years of market price data used for each. index.